Kinetic Theory of Gases

- Theory describing the behavior of an ideal gas.

Assumptions

1) A container may contain a large number (N) of identical molecules with mass (m)

   - molecule size << molecular separation
   - molecule size infinitesimall compared to container size

2) Molecules move randomly and obey Newton's Laws.

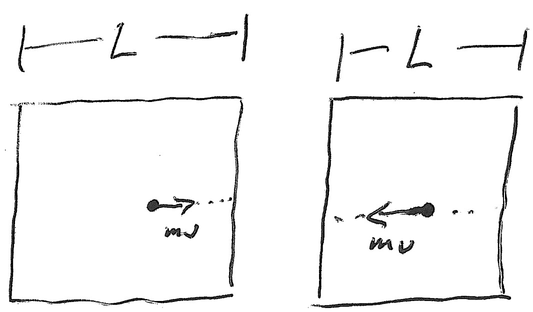
3) The only interactions between molecules and with the container are elastic collisions

Description of Pressure

- molecules with momentum mv collide with container walls.

Elastic collision (in 1D space)

$$\Delta p = 2mv$$



time to bounce off the same wall

$$\Delta t = \frac{2L}{v}$$

m = mass
L = length
v = speed
A = surface area
V = volume

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{2L}{v}} = \frac{mv^2}{L}$$

p = momentum
P = pressure

$$P = \frac{F}{A} = \frac{\frac{mv^2}{L}}{L^2} = \frac{mv^2}{L^3} = \frac{mv^2}{V}$$

Speed Distribution of Molecules

Individual molecules in 1-dimension

$$P = \frac{m\bar{v}^2}{V}$$

For all molecules (N)

$N = \#$ molecules

$$P = N\left(\frac{m\bar{v}^2}{V}\right)$$

$$\bar{V_x}^2 = \bar{v_y}^2 = \bar{v_z}^2$$

for a unit volume the 1-d model will work in any dimension

$$\bar{V}_{3d}^2 = \bar{v_x}^2 + \bar{v_y}^2 + \bar{v_z}^2$$

$$\bar{V_x}^2 = \frac{1}{3}\bar{V}_{3d}^2$$

Pressure for all molecules in 3-d

$$P = \left(\frac{N}{V}\right) m \, \frac{1}{3} \, \bar{v}_{3d}^{\,2}$$

Note: $E_k = \frac{1}{2} m v^2$

$$P = \frac{1}{3}\left(\frac{N}{V}\right) 2 E_K = \frac{2}{3}\left(\frac{N}{V}\right) \frac{1}{2} m \bar{v}_{3D}^{\,2}$$

or

$$PV = \frac{2}{3} N \, \frac{1}{2} m \bar{v}_{3D}^{\,2}$$

Ideal Gas Law

$$PV = NkT$$

so

$$\cancel{N} kT = \frac{2}{3} \cancel{N} \, \frac{1}{2} m \bar{v}_{3D}^{\,2}$$

or

$$\bar{E}_k = \frac{3}{2} kT$$

$$V_{RMS} = \sqrt{\bar{v}^2}$$

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

M ~~M~~ = molecular mass

$$m = \frac{M}{N_A}$$

$N_A = 6.02E23$

m = individual mass

$$N_A k = R$$

Internal Energy (U)

$$U = \frac{3}{2} NkT$$

total kinetic energy of all molecules

$$U = \frac{3}{2} nRT$$