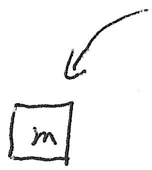
Center of Mass

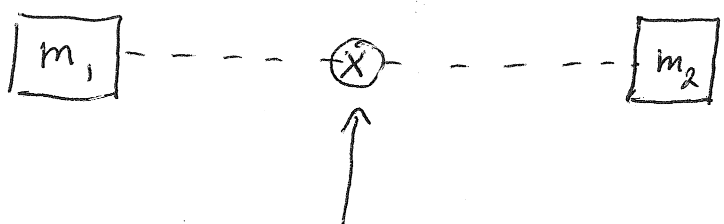
Center of mass involves the way mass is distributed in a system.

* For a single mass system that is very small, the object is treated as a point where all the mass is concentrated.

Center of mass is the object



* For a two-mass system the center of mass does not need to be within the object

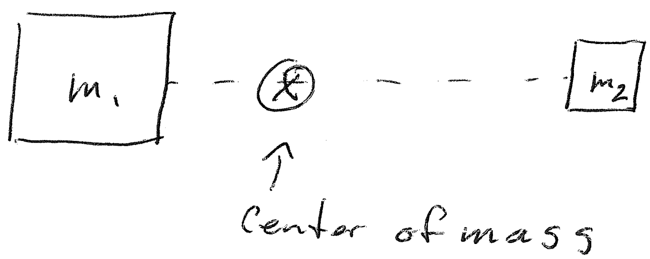


Center of mass

If $m_1 = m_2$

the center of mass is half way between the masses.

If $m_1 > m_2$ then we must recognize that center of mass moves toward the larger object



This particular two object system can be thought of as a balancing act.

To balance the system a support must be placed closer to the large object.

def Center of Mass

1) - Point where a system can be balanced in a uniform gravitational field.

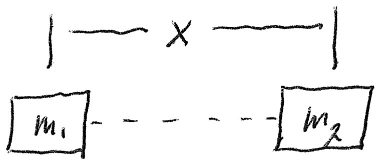
2) - Point where all of the mass of a system is concentrated for purposes of calculation.

Equation

1) 1-D Center of mass

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

Ex 1



$x = 11\,cm$
$m_1 = 250\,g$
$m_2 = 450\,g$

Determine the center of mass of the system pictured above.

Let m_1 be located at 0 cm $\Rightarrow x_1 = 0\,cm$

and m_2 located at 11cm $\Rightarrow x_2 = 11\,cm$

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{(.250\,kg)(0\,m) + (.450\,kg)(0.11\,m)}{(0.250\,kg + 0.450\,kg)}$$

$$= 0.071\,m$$

$$\boxed{X_{cm} = 7.1\,cm}$$

Notice that the locations of the masses in the system were given relative to an origin.

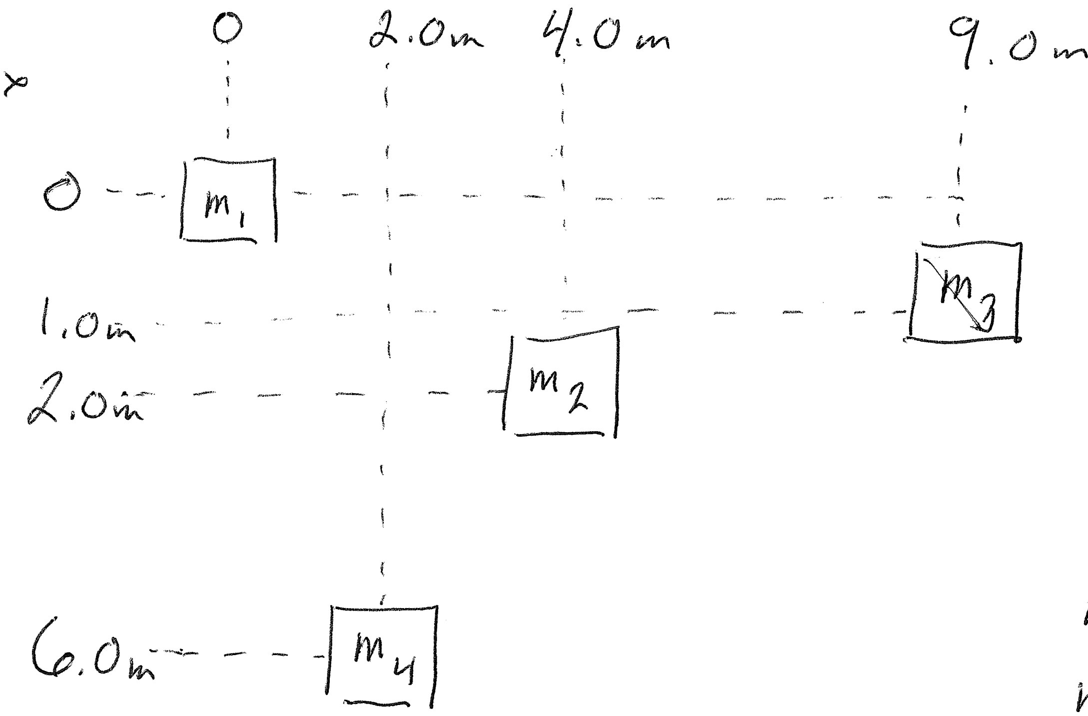
You can choose the origin to suit the problem situation. Just make sure all positions are reported relative to the origin you define.

To expand center of mass to two dimension simply account for another dimension.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\Sigma m x}{M_{system}}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\Sigma m y}{M_{system}}$$

Ex



$m_1 = 10.\text{kg}$

$m_2 = 12.\text{kg}$

$m_3 = 9.0\text{kg}$

$m_4 = 6.0\text{kg}$

Determine the center of mass of the system.

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{10.\text{kg}(0\text{m}) + 12.\text{kg}(4.0\text{m}) + 9.0\text{kg}(9.0\text{m}) + 6.0\text{kg}(2.0\text{m})}{10.\text{kg} + 12.\text{kg} + 9.0\text{kg} + 6.0\text{kg}}$$

$$= \boxed{3.8\text{ m}}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{10.\,kg(0m) + 12\,kg(2.0m) + 9.0kg(1.0m) + 6.0kg(6.0m)}{10.\,kg + 12\,kg + 9.0kg + 6.0kg}$$

$$= 1.86\,m$$

$$= \boxed{1.9m}$$

$(3.8m, 1.9m)$ center-of-mass

Motion and Center of Mass

Velocity of Center of Mass

$$V_{cm} = \frac{m_1 v_1 + m_2 V_2 + \dots}{m_1 + m_2 + \dots}$$

$$M_{system} V_{cm} = m_1 v_1 + m_2 V_2 + \dots = p_1 + p_2 + \dots = p_{system}$$

Acceleration of Center of Mass

$$A_{cm} = \frac{m_1 a_1 + m_2 A_2 + \dots}{m_1 + m_2 + \dots}$$

$$M_{system} A_{cm} = m_1 a_1 + m_2 a_2 + \dots = F_1 + F_2 + \dots = F_{system}$$

Recall that

$$F_{Net} = ma_{system}$$

This also applies to center of mass.

If $F_{Net} = 0$ for external forces then the center of mass of a system will continue to exist in its original state of motion.