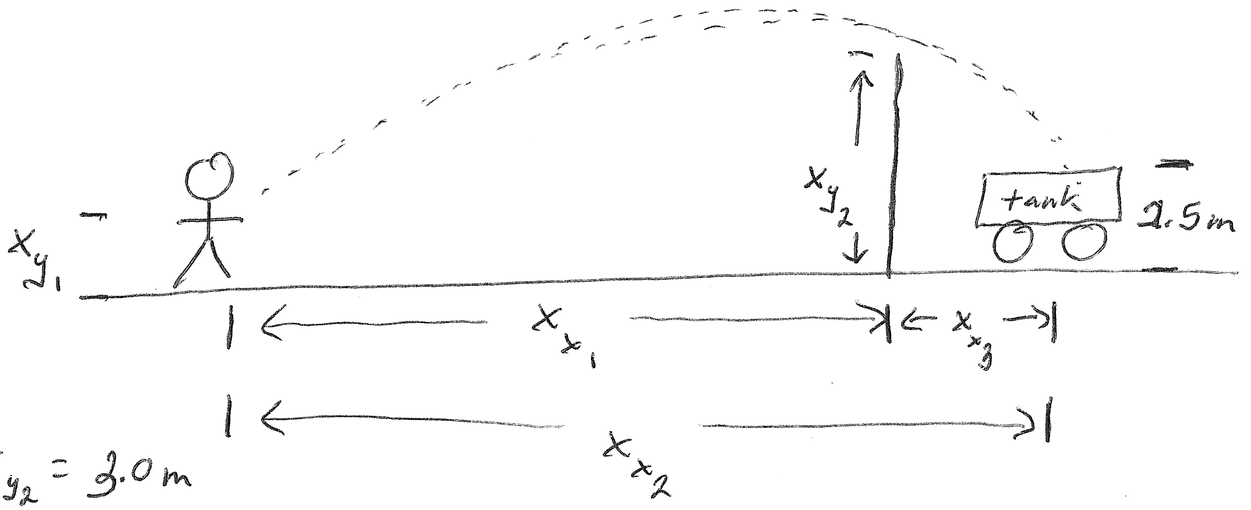
Example



$X_{y_2} = 3.0m$
$X_{y_1} = 1.0m$

$X_{x_1} = 25m$

$X_{x_2} = 28m$

$X_{x_3} = 3.0m$

Determine the launch velocity required to hit a row of tanks placed behind a wall as indicated above. Note: the projectile must clear the fence and $V_0 = 28 m/s$.

x-dimension

$V = V_0 \cos \Theta$
$x_0 = 0$
$x = 28m$
$\Delta t = ?$

y-dimension

$V_{0_y} = V_0 \sin \Theta$
$x_0 = 1.0m$
$x = 2.5m$
$a = -9.81 m/s^2$
$\Delta t = ?$

x-dimension

$$x = \overset{0}{\cancel{x_0}} + V\Delta t$$

$$28m = V_0 \cos\theta \, \Delta t$$

$$\Delta t = \frac{28m}{V_0 \cos\theta}$$

y-dimensson

$$x = x_0 + V_0 \Delta t + \frac{1}{2} a \Delta t^2$$

$$2.5m = 1.0m + V_0 \sin\theta \, \Delta t + \frac{1}{2}\left(-9.81 m/s\right)\Delta t^2$$

$$2.5m = 1.0m + \cancel{V_0} \sin\theta \left(\frac{28m}{\cancel{V_0} \cos\theta}\right) + \frac{1}{2}\left(-9.81 m/s^2\right)\left(\frac{28m}{V_0 \cos\theta}\right)^2$$

$$2.5m = 1.0m + \left(\frac{\sin\theta}{\cos\theta}\right) 28m + \left[\frac{\frac{1}{2}\left(-9.81 m/s^2\right)\cancel{(28m)^2}}{\cancel{\left(28 m/s\right)^2}}\right]\frac{1}{\cos^2\theta}$$

$$2.5m = 1.0m + 28m \tan\theta + \left(-4.9 m\right)\left(\tan^2\theta + 1\right)$$

<u>Look it up or derive it</u>

$$\tan^2\theta + 1 = \frac{1}{\cos^2\theta}$$

$0 = -1.5m + 28m \text{ Tan } \theta + (-4.9m) \text{ Tan}^2 \theta + (-4.9m)$

$0 = -6.4m + 28m \text{ Tan } \theta + (-4.9m) \text{ Tan}^2 \theta$

Use quadratic equation for $\tan \theta$

$\text{Tan } \theta = 0.2835$

5.4758

angle 1 $\theta = \tan^{-1}(0.2835) = 13.4°$

angle 2 $\theta = \tan^{-1}(5.4758) = 79.7°$

Now check the fence.

angle 1

y-components

$V_0 = 28 \text{ m/s} \sin 13.4° = 6.49 \text{ m/s}$

$a = -9.81 \text{ m/s}^2$

$x_0 = 0 \text{ m}$

$x =$ at or above 3.0m

x-components

$V_0 = 28 \text{ m/s} \cos 13.4°$

$= 27.24 \text{ m/s}$

$x_0 = 0$

$x = 25m$

y-components

$$x = \overset{0}{\cancel{x_0}} + v_0 \Delta t + \tfrac{1}{2} a \Delta t^2$$

x-components

$$x = x_0 + v_0 \Delta t$$

$$\Delta t = \frac{x - x_0}{v_0} = \frac{25\text{m} - 0\text{m}}{27.4\,\text{m/s}}$$

$$= 0.9124\text{s}$$

$$x = 6.49\,\text{m/s}\,(0.9124\text{s}) + \tfrac{1}{2}\left(-9.81\,\text{m/s}^2\right)\left(0.9124\text{s}\right)^2$$

$$= \underline{1.838\text{m}}$$

$1.838\text{m} < 3.0\text{m}$

Hits the fence!

angle 2

y-components

$V_0 = 28\,m/s \sin 79.7^\circ$

$= 27.55\,m/s$

$a = -9.81\,m/s^2$

$x_0 = 0$

$x =$ at or above 3.0 m

x-components

$V = 28\,m/s \cos 79.7^\circ$

$= 5.01\,m/s$

$x_0 = 0\,m$

$x = 25\,m$

$$x = x_0 + V_0 \Delta t$$

$$\Delta t = \frac{x - x_0}{v_0} = \frac{25\,m - 0\,m}{5.01\,m/s}$$

$$= \underline{4.9900\,s}$$

$$x = \overset{0}{\cancel{x_0}} + v_0 \Delta t + \tfrac{1}{2} a \Delta t^2$$

$$x = 27.55\,m/s\,(4.9900\,s) + \tfrac{1}{2}(-9.81\,m/s^2)(4.9900\,s)^2$$

$= 15.34\,m$

$15.34\,m > 3.0\,m$

Wall is missed!

Launch velocity

$\boxed{28\,m/s \text{ @ } 79.7^\circ \text{ above horizontal}}$