60 A bicyclist is finishing his repair of a flat tire when a friend rides by with a constant speed of $3.5\ m/s$. Two seconds later the bicyclist hops on his bike and accelerates at $2.4\ m/s^2$ until he catches his friend.

(1) How much time does it take until he catches his friend?

(2) How far has he traveled?

(3) What is his speed when they meet?

(1) solution

bicyclist

$\Delta t = ?$

$v_i = 0\ m/s$

$a = 2.4\ m/s^2$

$x_i = 0\ m$

$$x_f = x_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$$

Write an equation describing the motion of each person

Note: because of the 2.0 s delay, the friend is ahead of the bicyclist.

Bicyclist

$\Delta t = ?$

$v_i = 0\ m/s$

$a = 2.4\ m/s$

$x_i = 0\ m$

Friend

$x_f = ?$

$x_i = 7.0\ m$

$v_i = 3.5\ m/s$

$\Delta t = ?$

equation describing

$$x_f = x_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$$

1st 2.0 s

Position change

$$\Delta x = (2.0\ s)(3.5\ m/s)$$

$$= 7.0\ m$$

$$x_f = 0 + 0\,\Delta t + \frac{1}{2}(2.4\ m/s^2)\,\Delta t^2$$

$$x_f = 1.2\ m/s^2\ \Delta t^2$$

$$x_f = 7.0\ m + 3.5\ m/s\ \Delta t + 0\,\Delta$$

$$x_f = 7.0\ m + 3.5\ m/s\ \Delta t$$

Find the intersection of the equations. (they both have Δt and x_f)

$$1.2\ m/s^2\ \Delta t^2 = 7.0\ m + 3.5\ m/s\ \Delta t$$

$$0 = \underbrace{7.0\ m}_{c} + \underbrace{3.5\ m/s}_{b}\ \Delta t + \underbrace{(-1.2\ m/s^2)}_{a}\ \Delta t^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-3.5\ m/s \pm \sqrt{(3.5\ m/s)^2 - 4(-1.2\ m/s^2)(7.0\ m)}}{2(-1.2\ m/s^2)}$$

$= 4.28\ s$ ← only this one makes sense in the problem

$= -1.36\ s$

$$\boxed{4.3\ s}$$

(2)

$$x_f = 1.2\ m/s^2\ \Delta t^2 \qquad \Delta t = 4.28\ s \quad \text{from (1)}$$

$$x_f = 1.2\ m/s^2 (4.28\ s)^2 = 21.98\ m$$

$$\boxed{22\ m}$$

(3)

$a = 2.4\ m/s^2$

$v_i = 0\ m/s$

$v_f = ?$

$\Delta x = 21.98\ m$ [from (2)]

$\Delta t = 4.28\ s$ [from (1)]

$$v_f = \cancel{v_i}^{\,0} + a\Delta t$$

$$v_f = 2.4\ m/s^2\ 4.28\ s$$

$$= 10.27\ m/s$$

$$\boxed{10.\ m/s}$$