Uniform Circular Motion
and
Simple Harmonic Motion

Angular Position (θ)

$$\theta = \omega t$$

ω = angular speed (rad/s)

t = time (s)

In SHM ω is angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

SHM position (x)

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

$$x = A\cos(\omega t)$$ using angular frequency from above

SHM velocity

$$V = -A\omega \sin(\omega t)$$

Note: maximum velocity occurs when

$$\sin(\omega t) = 1$$

$$|V_{max}| = A\omega \qquad \text{(or maximum speed)}$$

SHM Acceleration

$$a = -A\omega^2 \cos(\omega t)$$

Note: maximum acceleration occurs when $\cos(\omega t) = 1$

$$|a_{max}| = A\omega^2$$

(magnitude of or maximum acceleration)

Spring-Mass Systems and Energy

Period (T) for a spring-mass system

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Angular frequency (ω) for a spring-mass system

$$\omega = \sqrt{\frac{k}{m}}$$

Energy Storage

$$E_{Total} = E_k + E_{el}$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$x = A \cos(\omega t)$$

$$v = -A\omega \sin(\omega t)$$

$$E_{Total} = \frac{1}{2} m \left(-A\omega \sin(\omega t)\right)^2 + \frac{1}{2} k \left(A \cos \omega t\right)^2$$

$$= \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t) + \frac{1}{2} k A^2 \cos^2(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 = \frac{k}{m}$$

for spring-mass system

$$= \frac{1}{2} \cancel{m} A^2 \left(\frac{k}{\cancel{m}}\right) \sin^2(\omega t) + \frac{1}{2} k A^2 \cos^2(\omega t)$$

$$= \frac{1}{2} k A^2 \left(\sin^2(\omega t) + \cos^2(\omega t)\right)$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\boxed{E_{Total} = \frac{1}{2} k A^2}$$