An object is allowed to fall vertically through a fluid where the drag force is described by $F_{drag} = kV$

Suppose the object is released from rest and it experiences negligible buoyant force.

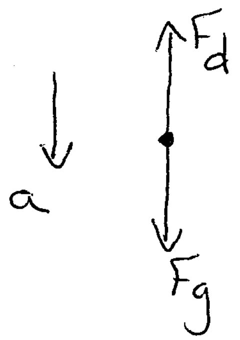
(1) Draw a force diagram of the object

(2) Write a differential equation describing the motion of the object

(3) Determine the terminal velocity of the object.

(4) Solve the differential equation.

1



$$\Sigma F_y = -ma = F_d + -F_g$$

$$F_d = kv$$
$$F_g = mg$$

$$-ma = kv - mg$$

2

$$a = -\frac{k}{m}v + \cancel{m}g \qquad a = \frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = -\frac{k}{m}v + \cancel{m}g}$$

3 Terminal velocity $\Rightarrow \Sigma F_y = 0$

$$0 = -\frac{k}{m}v + \cancel{m}g$$

$$\frac{k}{m}v = \cancel{m}g$$

$$v = \frac{\cancel{m}g}{\left(\frac{k}{m}\right)}$$

$$\boxed{v = \frac{mg}{k}}$$

(4) Use separation of variables

$$\frac{dv}{dt} = -\frac{k}{m}v + g$$

$$\frac{dv}{-\frac{k}{m}v+g} = dt$$

$$\int_0^v \frac{dv}{-\frac{k}{m}v+g} = \int_0^t dt$$

<u>u-substitution</u> $= t$

$u = -\frac{k}{m}v + g$

$du = -\frac{k}{m}\,dv$

<u>Limits</u>

$v \to -\frac{k}{m}v+g$

$0 \to g$

$$\int_{g}^{-\frac{k}{m}v+g} \frac{-\frac{m}{k}\,du}{u} = t$$

$$\int_{g}^{-\frac{k}{m}v+g} \frac{du}{u} = -\frac{k}{m}t$$

$$\ln u \Big|_{g}^{-\frac{k}{m}v+g} = -\frac{k}{m}t$$

$$\ln\left(-\frac{k}{m}v+g\right) - \ln(g) = -\frac{k}{m}t$$

$$\ln\left(\frac{-\frac{k}{m}v+g}{g}\right) = -\frac{k}{m}t$$

$$e^{\left[\ln\left(\frac{-\frac{k}{m}v+g}{g}\right)\right]} = e^{\left[-\frac{k}{m}t\right]}$$

$$\frac{-\frac{k}{m}v+g}{g} = e^{-\frac{k}{m}t}$$

$$-\frac{k}{m}v+g = g\,e^{-\frac{k}{m}t}$$

$$-\frac{k}{m}v = -g + g\,e^{-\frac{k}{m}t}$$

$$-\frac{k}{m}v = -g\left(1-e^{-\frac{k}{m}t}\right)$$

$$V = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)$$