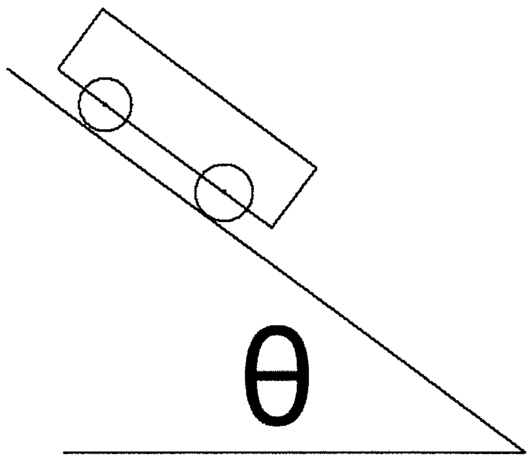
Cart on an Incline with Drag Force

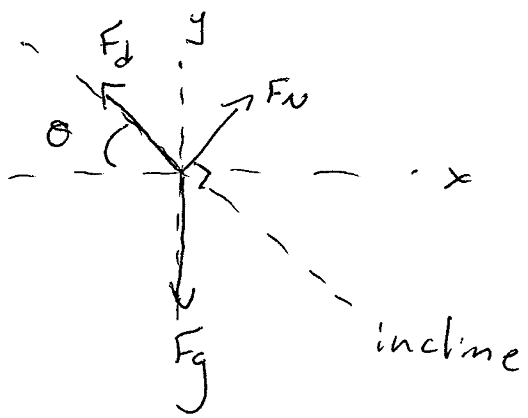
A cart is released along an incline as indicated in the figure. A drag force described by $F_{drag} = kv$ is the only force that acts to slow the cart along the incline.



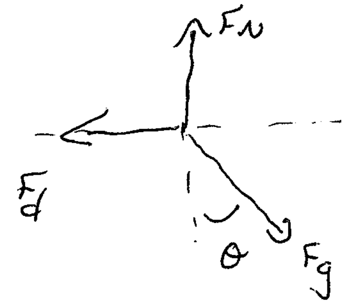
1. Write an equation that describes the terminal velocity the cart would reach moving down the incline.
2. Write a differential equation that describes the motion of the cart moving down the incline. Do not solve.
3. Solve the differential equation from the previous part to write an equation that describes the velocity of the cart moving down the incline. Assume that the cart was initially at rest and time started when the cart was released. In other words; $x_0 = 0$ and $t_0 = 0$

(1)

1st step: Draw force diagrams



rotate for convenience



2nd step: Write equations using rotated force diagram

$$\Sigma F_x = Ma = -F_d + F_g \sin\theta$$

$$\Sigma F_y = 0 = F_N + -F_g \cos\theta$$

3rd step: Note that ΣF_x is the only portion that involves the quantity of interest

$$F_d = kv \qquad F_g = Mg$$

$$Ma = -kv + Mg \sin\theta$$

(1) continued

4th step: What does terminal velocity mean?

In our case it means $\Sigma F_x = 0$

$$0 = -kv + Mg\sin\theta$$

$$kv = Mg\sin\theta$$

$$\boxed{v = \frac{Mg\sin\theta}{k}}$$

This is the expression for terminal velocity.

(2)

Start with the result from step 3 in part 1.

$$Ma = -kv + Mg\sin\theta$$

Recall

$$a = \frac{dv}{dt}$$

$$M\frac{dv}{dt} = -kv + Mg\sin\theta$$

or

$$\boxed{\frac{dv}{dt} = \frac{-k}{M}v + g\sin\theta}$$

(3)

Start with the result from part 2.

$$\frac{dv}{dt} = \frac{-k}{M} v + g\sin\theta$$

Step 1: separate the variables to integrate

$$\int_0^V \frac{dv}{\frac{-k}{M} v + g\sin\theta} = \int_0^t dt$$

Step 2: Use u-substitution to simplify your problem notation

$$u = -\frac{k}{M} v + g\sin\theta$$

$$du = -\frac{k}{M} dv$$

(3) - continued

Step 3: Examine your limits

time is evaluated from 0 to t

Velocity is evaluated from 0 to v

↓

u-substitution may change limits. In this case it does.

* Note

don't forget to adjust for du

$$0 \to -\frac{k}{M}(0) + g\sin\theta$$

$$\to g\sin\theta$$

$$v \to -\frac{k}{M}v + g\sin\theta$$

Step 4: Rewrite your expression with substitution and new limits

$$\int_{g\sin\theta}^{-\frac{k}{m}v + g\sin\theta} -\frac{M}{k}\,\frac{du}{u} = \int_0^t dt$$

(3) - continued

Step 5: Integrate

$$-\frac{M}{k}\int_{g\sin\theta}^{-\frac{k}{m}v+g\sin\theta}\frac{dv}{v} = \int_0^t dt$$

Recall

$$\int_a^b \frac{dv}{v} = \ln v\Big|_a^b$$

$$= \ln b - \ln a$$

$$= \ln\frac{b}{a}$$

$$-\frac{M}{k}\ln\left(\frac{-\frac{k}{M}v+g\sin\theta}{g\sin\theta}\right) = t$$

(3) - continued

Step 6: solve for v

$$-\frac{M}{K} \ln\left(\frac{-\frac{K}{M} v + g\sin\theta}{g\sin\theta}\right) = t$$

$$\ln\left(\frac{-\frac{K}{M} v + g\sin\theta}{g\sin\theta}\right) = -\frac{K}{M} t$$

$$\frac{-\frac{K}{M} v + g\sin\theta}{g\sin\theta} = e^{-\frac{K}{M} t}$$

$$-\frac{K}{M} v + g\sin\theta = g\sin\theta\, e^{-\frac{K}{M} t}$$

$$-\frac{K}{M} v = -g\sin\theta + g\sin\theta\, e^{-\frac{K}{M} t}$$

$$\frac{K}{M} v = g\sin\theta - g\sin\theta\, e^{-\frac{K}{M} t}$$

$$\boxed{v = \frac{M}{K} g\sin\theta \left(1 - e^{-\frac{K}{m} t}\right)}$$