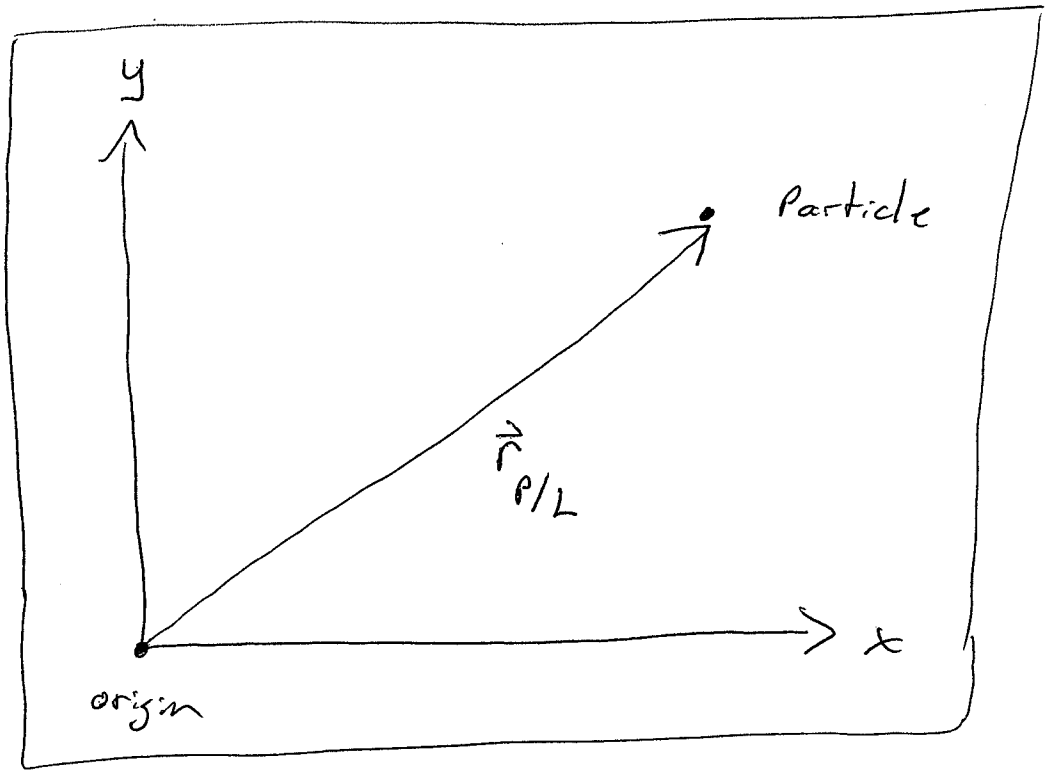
Relative Velocity Notes

Suppose you are making measurements of a particle in a laboratory.

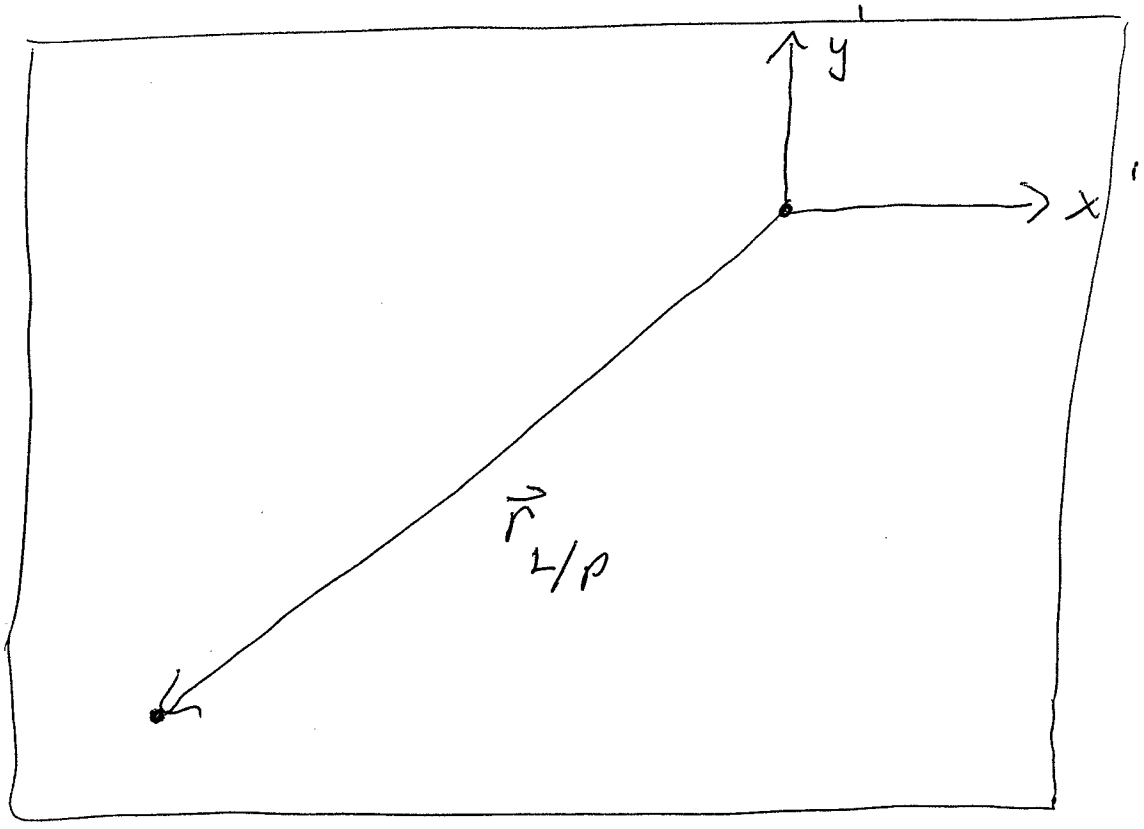
You first measure position by defining the lab (or some point in the lab) to be the origin.

This defines a "Laboratory Frame of Reference"



$\vec{r}_{P/L}$ = position of particle with respect to the lab.

You could just as easily use the particle as the origin and locate the same point of reference in the lab.



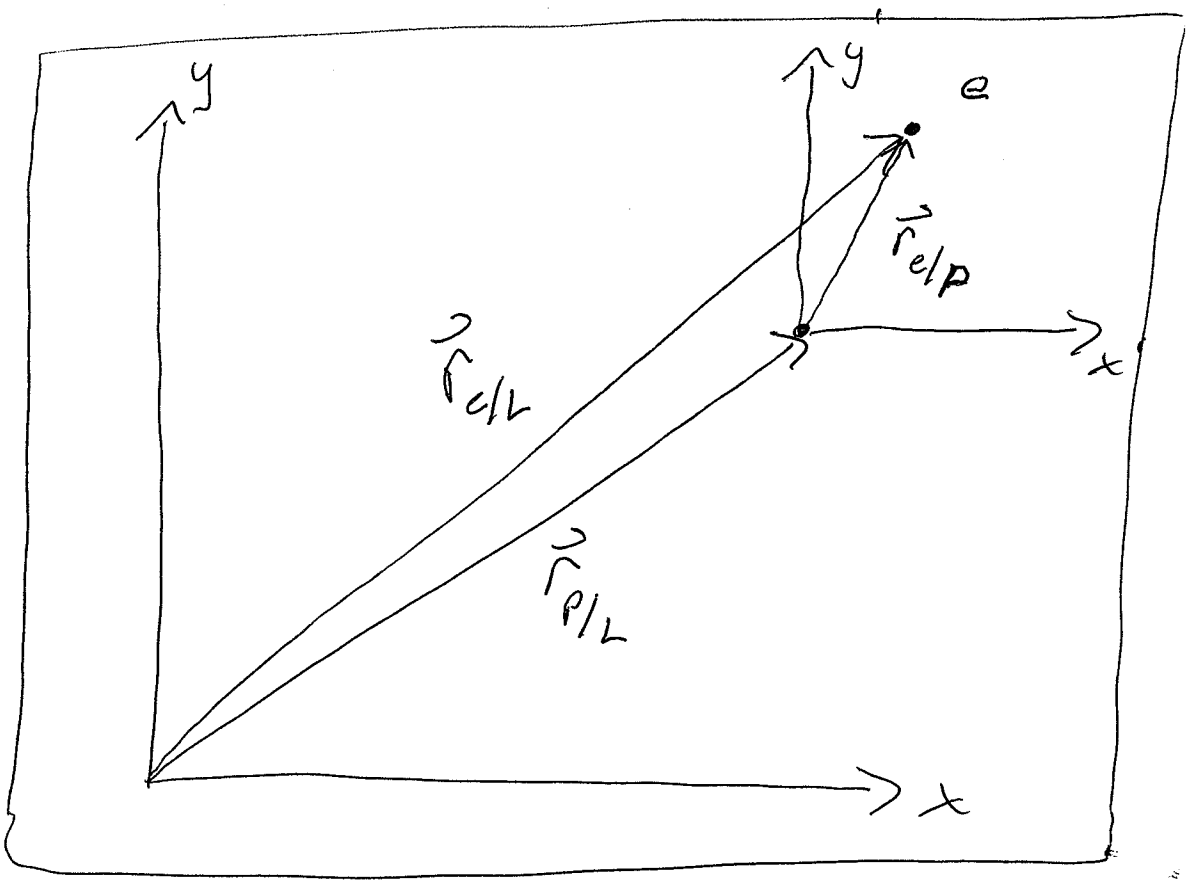
This defines a "Particle Frame of Reference"

$\vec{r}_{L/P}$ = position of lab relative to the particle

Notice that

$$\vec{r}_{P/L} = -\vec{r}_{L/P}$$

Now we can relate the two reference frames by thinking about some event (e).



Notice that

position of e with respect to P

$$\vec{r}_{e/L} = \vec{r}_{e/p} + \vec{r}_{p/L}$$

position of P with respect to L

position of e with respect to L

Easy way to remember formula is to match the "inside" subscripts

The derivative is a linear operator

so

$$\frac{d\,\vec{r}_{e/L}}{dt} = \frac{d\,\vec{r}_{e/p}}{dt} + \frac{d\,\vec{r}_{p/L}}{dt}$$

or

$$\vec{v}_{e/L} = \vec{v}_{e/p} + \vec{v}_{p/L}$$