Vectors

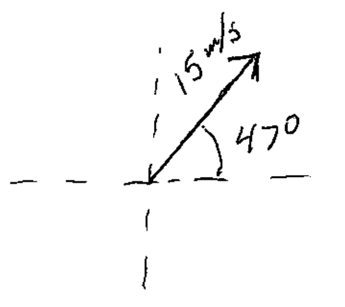
Vector

- a quantity representing magnitude and direction

Examples:
displacement
velocity
acceleration
force

Graphical Representation

- directed line segment



Component Representation

$$\vec{V} = -11\,\text{m/s}\ \vec{i} + 3.1\,\text{m/s}\ \vec{j} + 1.0\,\text{m/s}\ \vec{k}$$

Magnitude-Angle Representation

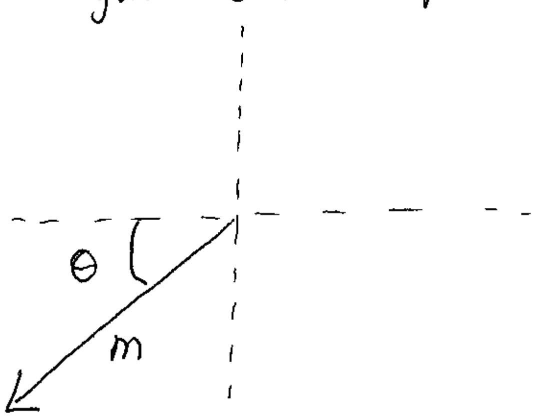
$$\vec{V} = 27\,\text{m} \ @\ 27^\circ$$

Except for graphical methods, vector math operations are defined for vectors in component form.

You will need to be able to switch from/to angle-magnitude representation.

General Example:

angle-magnitude to components



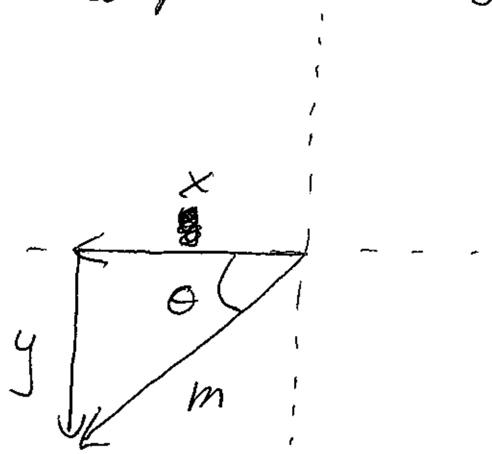
$\sin\theta = \frac{y}{m}$

$y = m \sin\theta$

$\cos\theta = \frac{x}{m}$

$x = m\cos\theta$

Components to angle-magnitude

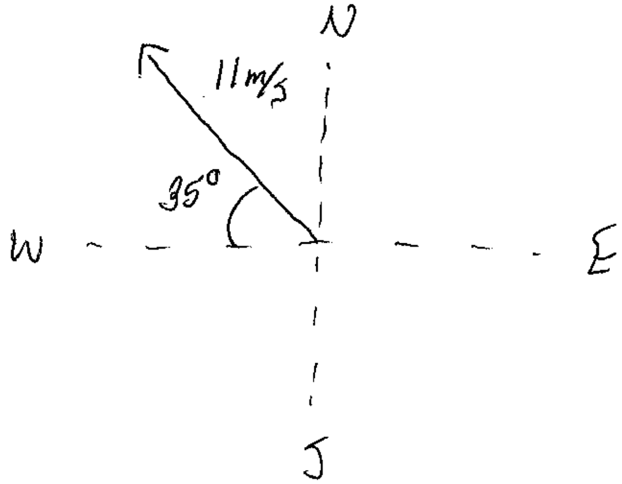


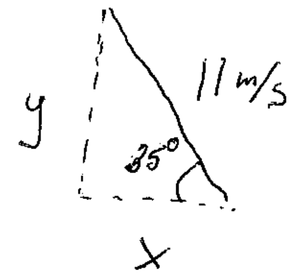
$m = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Number Example: $11\,m/s$ @ $35°$ N of W





$$\sin 35° = \frac{y}{11\,m/s}$$

$$y = 11\,m/s \sin 35° = 6.3\,m/s$$

$$\cos 35° = \frac{x}{11\,m/s}$$

$$x = 11\,m/s \cos 35° = 9.0\,m/s$$

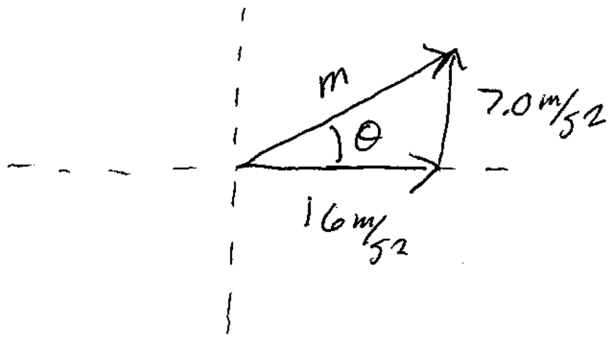
$$9.0\,m/s\ \vec{x} + 6.3\,m/s\ \vec{y}$$

↑

Notation Note: this could be written as $9.0\,m/s\ \vec{i} + 6.3\,m/s\ \vec{j}$

Number Example: Component to angle-magnitude

$$16\,\mathrm{m/s^2}\,\vec{x} + 7.0\,\mathrm{m/s^2}\,\vec{y}$$



$$\tan\theta = \frac{7.0\,\mathrm{m/s^2}}{16\,\mathrm{m/s^2}}$$

$$\theta = \tan^{-1}\left(\frac{7.0}{16}\right) = 23.6^\circ$$

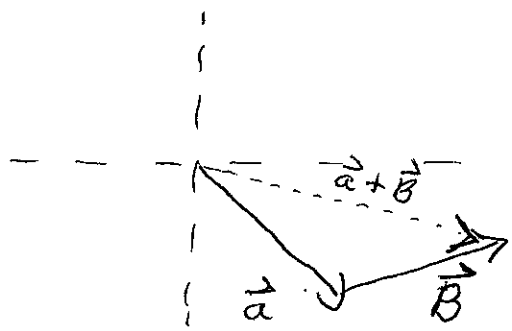
$$m = \sqrt{\left(7.0\,\mathrm{m/s^2}\right)^2 + \left(16\,\mathrm{m/s^2}\right)^2} = 17.5\,\mathrm{m/s^2}$$

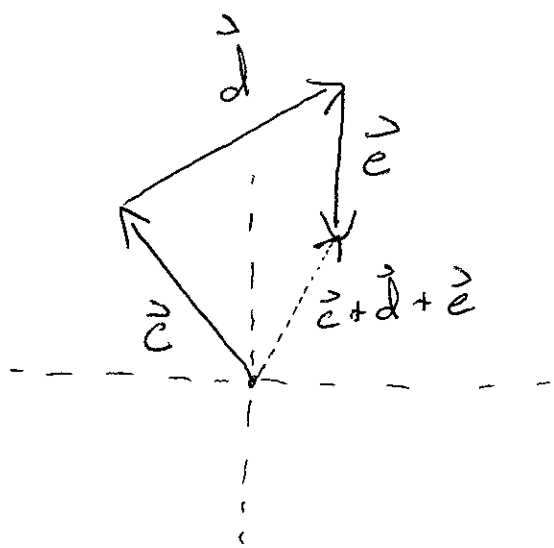
Vector Addition

Graphical Method

- place vectors head-to tail and draw in resultant.

Ex





Component Method

- add like components

$$\vec{a} = 5\,m/s\;\vec{x} + 7\,m/s\;\vec{y}$$

$$\vec{b} = -2\,m/s\;\vec{x} + 4\,m/s\;\vec{y}$$

	$\vec{x}$	$\vec{y}$
$\vec{a}$	5 m/s	7 m/s
+		
$\vec{b}$	−2 m/s	4 m/s
$\vec{a}+\vec{b}$	$3\,m/s\;\vec{x}$ +	$11\,m/s\;\vec{y}$

Vector Multiplication

Three different ways

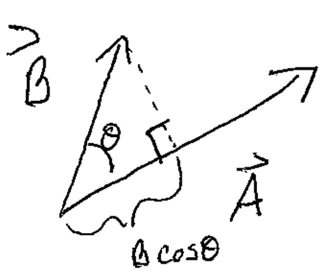
(1) Scalar (Dot) Product (two vectors)

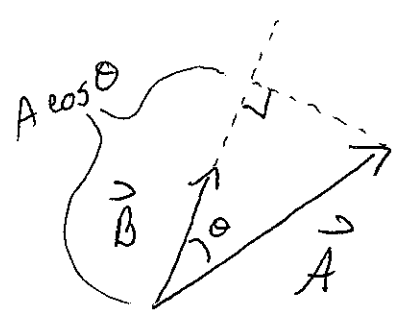
(2) Vector (Cross) Product (two vectors)

(3) Multiply a vector by a scalar (only one vector)

Dot Product

- multiplication of a vector and a part of another vector that acts in the same direction





$\vec{A} \cdot \vec{B} = A\,(B\cos\theta)$

$\vec{B} \cdot \vec{A} = B\,(A\cos\theta)$

* Produces a scalar result!

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Cross Product

$$\vec{A} \times \vec{B} = \underbrace{AB\sin\theta}_{\text{magnitude}} \; \vec{n}$$

($\vec{n}$ is a unit vector defined by the right-hand rule)

$\vec{n}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{B}$

In our case:

* $\vec{n}$ for $\vec{A} \times \vec{B}$ is out of the page
* $\vec{n}$ for $\vec{B} \times \vec{A}$ is into the page

Cross Product continued

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\,\vec{i} + (A_z B_x - A_x B_z)\,\vec{j} + (A_x B_y - A_y B_x)\,\vec{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \vec{i} + \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \vec{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \vec{k}$$