Torque ($\tau$) can be calculated using the equation $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ is the position vector locating the point the force $\vec{F}$ is applied from the axis of rotation. Suppose a force $\vec{F} = (2.7\,N)\vec{x} + (3.3\,N)\vec{y} + (-2.4\,N)\vec{z}$ is applied to an irregularly shaped object at $\vec{r} = (-0.44\,m)\vec{x} + (1.0\,m)\vec{y} + (-0.97\,m)\vec{z}$.

(1) Determine the torque produced.

(2) Determine the angle between $\vec{r}$ and $\vec{F}$.

(1) Solution

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\,\vec{x} + (A_z B_x - A_x B_z)\,\vec{y} + (A_x B_y - A_y B_x)\,\vec{z}$$

In our case we have

$\vec{r} \times \vec{F}$ rather than $\vec{A} \times \vec{B}$

$$\vec{r} \times \vec{F} = \left[(1.0\,m)(-2.4\,N) - (-0.97\,m)(3.3\,N)\right] \vec{x}$$

$$\left[(-0.97\,m)(2.7\,N) - (-0.44\,m)(-2.4\,N)\right] \vec{y}$$

$$\left[(-0.44\,m)(3.3\,N) - (1.0\,m)(2.7\,N)\right] \vec{z}$$

$$\boxed{\vec{\tau} = (0.801\,mN)\,\vec{x} + (-3.68\,mN)\,\vec{y} + (-4.152\,mN)\,\vec{z}}$$

(2) solution

Use the dot product

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\theta = \cos^{-1}\left(\frac{A_x B_x + A_y B_y + A_z B_z}{A\,B}\right)$$

Let $\vec{r} = \vec{A}$
$\vec{F} = \vec{B}$

Then $r = A = \sqrt{(-0.44m)^2 + (1.0m)^2 + (-0.97m)^2}$

$= 1.46m$

$F = B = \sqrt{(2.7N)^2 + (3.3N)^2 + (-2.4N)^2}$

$= 4.89N$

$$\theta = \cos^{-1}\left(\frac{(-0.44)(2.7) + (1.0)(3.3) + (-0.97)(-2.4)}{(1.46)(4.89)}\right)$$

$\boxed{= 51.5^\circ}$

$\boxed{52^\circ}$ 2-sig fig