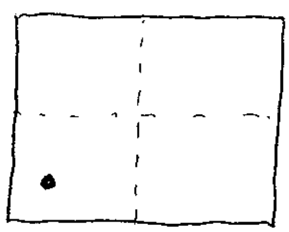
# Kinematics

Kinematics is the study of motion without regard to force or mass.

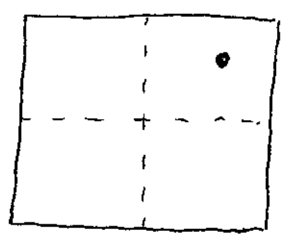
## Motion

- A change in the position of a particle observed with respect to time.

<u>Ex</u> Suppose you look at the following two screen shots of an object on radar.



Initial Observation



Next Observation

According to your observations, the object was initially in quadrant 3. The next observation shows the object in quadrant 1.

Since the position of the object is different you say that the object moved, or that motion occurred.

Position (x)

- The location of an object relative to some defined origin.
- measured in meters (m)
- the variable x will be used in equations to represent position.

For our purposes, position will mostly be viewed in the 2D perspective. However, occasionally we will use the 1D and 3D perspectives as well.

In the 2D perspective a change in position can be viewed in two different ways.

(1) Strictly speaking a change in position ($\Delta x$) is simply the straight-line distance between the final ($x$) and initial ($x_0$) positions.

$$\Delta x = x - x_0$$

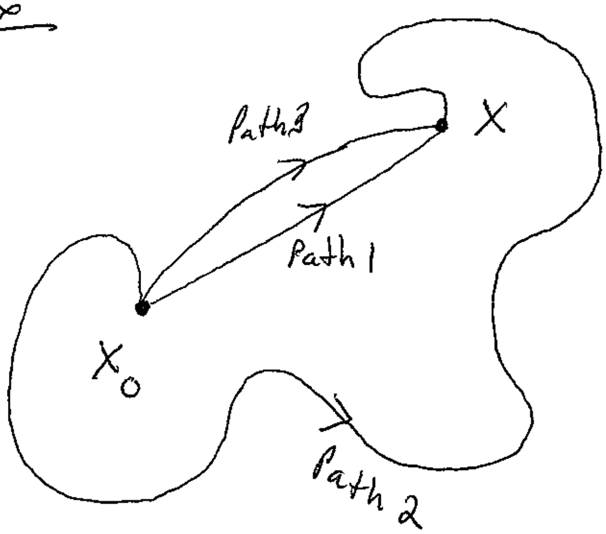
* This may be referred to as displacement

displacement and change in position will be used interchangeabl

(2) Sometimes the path between the initial and final positions is important.

In this case we are actually interested in more than the position change. We are interested in the length of the path between the initial and final points.

Ex




* Note here that path 1 is the only path that represents $\Delta x$.

* All other paths may produce the same $\Delta x$.

Once it is established that motion has occurred we are interested in quantifying that motion.

How far did it move?

How fast did it move?

In what direction did it move?

Was the motion constant?

How much does the motion change?

To quantify these questions we will use the following terms:

displacement ($\Delta x$)

- How far from $x_0$ to $x$ and in what direction.

velocity ($\Delta V$)

- How fast and in what direction.

acceleration ($\Delta a$)

- Was the motion constant? If not, in what direction was it changing and by how much?

Velocity ($v$)

- Descriptor of motion that quantifies how quickly and in what direction the position of an object is changing.
- We will use this as a descriptor of an object's state of motion.
- measured in meters per second ($\frac{m}{s}$)

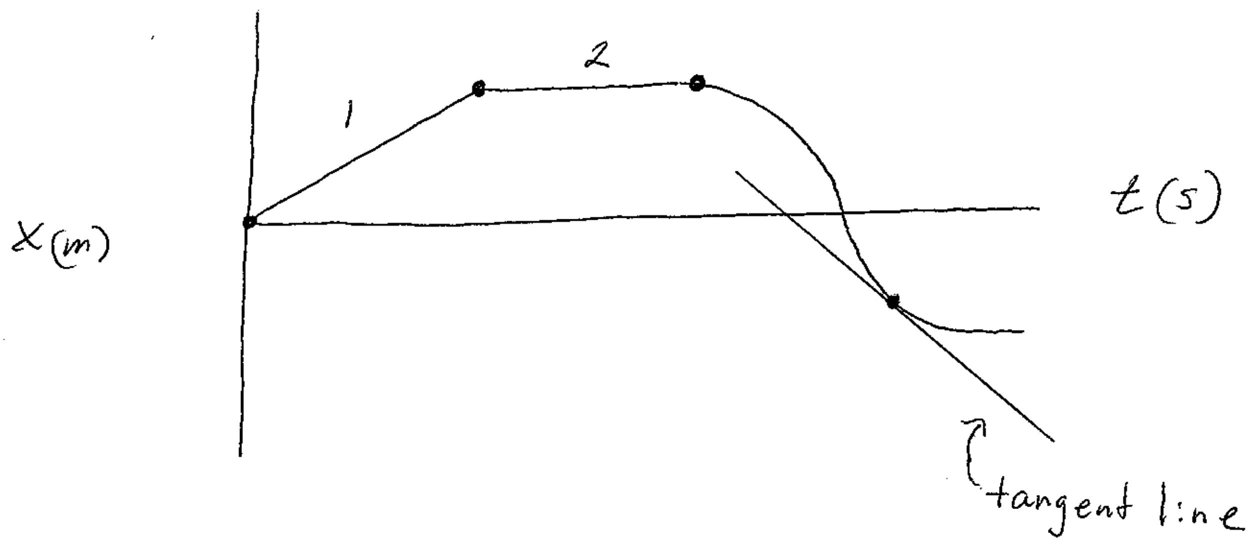
Average Velocity ($\Delta V$)

$$\Delta V = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0}$$

Instantaneous Velocity ($v$)

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

In graphical terms velocity is the slope of the position vs time graph.




* If the sections of x vs t are line segments you can simply use slope to determine velocity.

* If the sections of x vs t are curves you need to use the derivative. The graphical way to perform a derivative is to determine the slope of a line tangent to a curved segment at a particular point of interest.

# Acceleration (a)

- Descriptor of motion that quantifies how quickly and in what direction the velocity of the object is changing.

- we will use this as another descriptor of an object's state of motion.

- measured in meters per second per second $\left(\frac{m}{s^2}\right)$

## Average Acceleration ($\Delta a$)

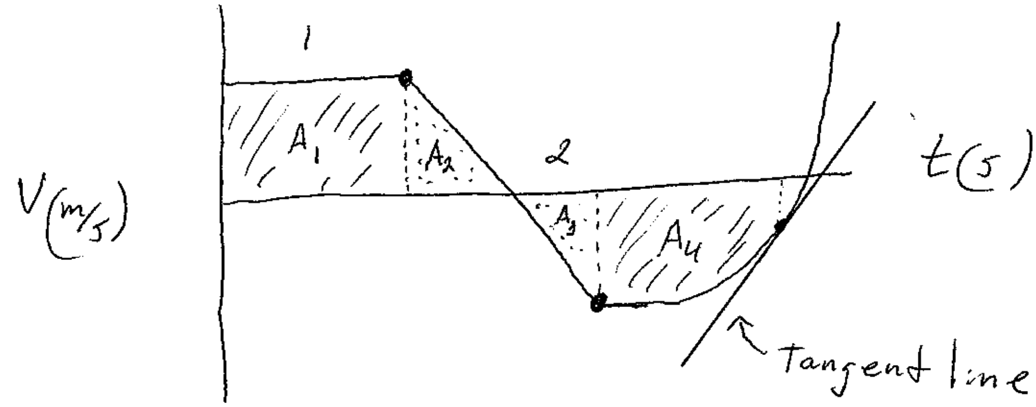
$$\Delta a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0}$$

## Instantaneous Acceleration (a)

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Note that: $\frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt}$

In graphical terms acceleration is the slope of the velocity vs time graph.



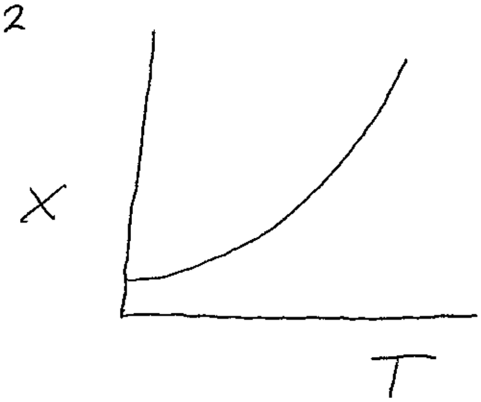

* If the sections of v vs t are line segments you can use slope to determine acceleration.

* If the sections curve, use the tangent line slope to estimate acceleration at that point.

* To find the position change for any given interval compute the area between the segment of v vs t and the t-axis.
  - $A_1$, $A_2$, $A_3$ are simple geometric shapes
  - $A_4$ requires calculus

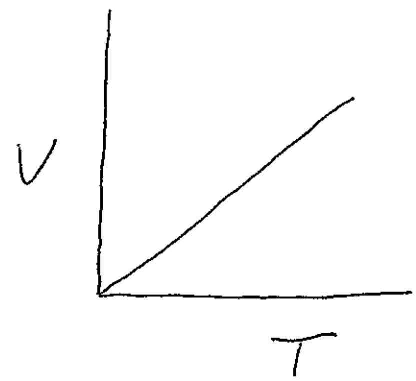
# Equations of Motion (constant a)

$x = x_0 + v_0 \Delta t + \frac{1}{2} a \Delta t^2$



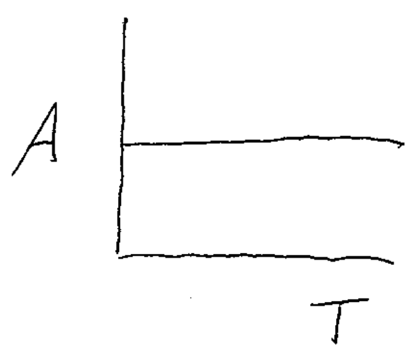
Characteristic graph $x(t)$

$v = v_0 + a \Delta t$



Characteristic graph $v(t)$

$v^2 = v_0^2 + 2a \Delta x$



Characteristic graph $a(t)$