# The Wave Equation



$$y(x,t) = f(x \pm vt)$$

Let $x \pm vt = z$

$$\Rightarrow y(x,t) = f(z)$$

$$z = x \pm vt$$

$$\frac{\partial z}{\partial x} = 1$$

$$\frac{\partial z}{\partial t} = \pm v$$

$$\frac{\partial y}{\partial x} = \frac{df}{dz}\frac{\partial z}{\partial x} = \frac{df}{dz}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{d\left(\frac{df}{dz}\right)}{dz}\frac{\partial z}{\partial x} = \frac{d^2 f}{dz^2}$$

$$\frac{\partial y}{\partial t} = \frac{df}{dz}\frac{\partial z}{\partial t} = \pm v \frac{df}{dz}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{d\left(\pm v \frac{df}{dz}\right)}{dz} \qquad \frac{\partial z}{\partial t} = \pm v \qquad = (\pm v)^2 \frac{d^2 f}{dz^2}$$

$$(\pm v)^2 = v^2$$

$$\frac{d^2 f}{dz^2} = \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}}$$

Wave Equation
General Form