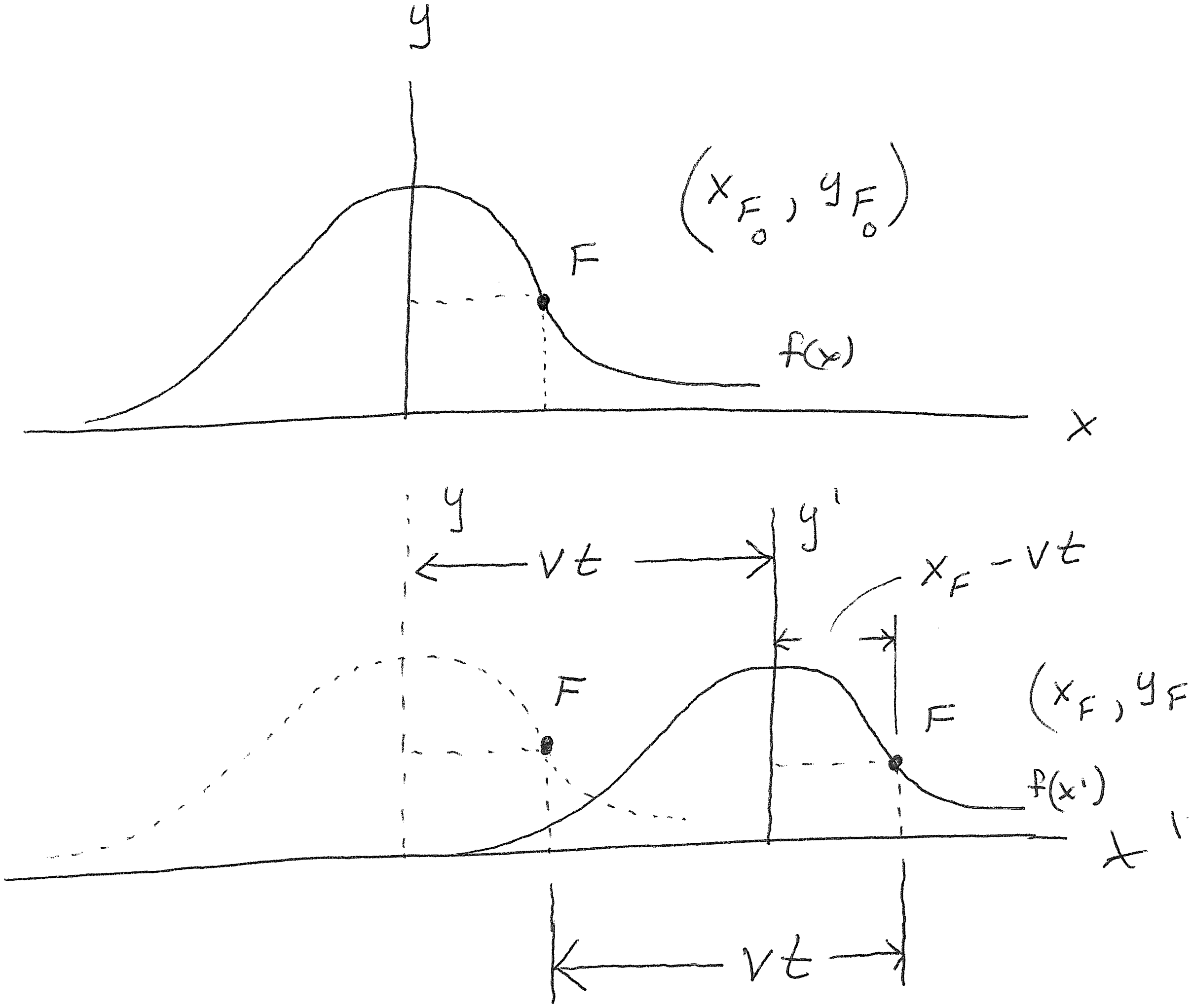
Mathematical Description of Waves



As a wave pulse travels through a medium without damping

$$y_{F_0} = y_F$$

* The wave shape does not change *

* Rather than shape change there is horizontal shift

x_F increases as the pulse travels right

$$x_F = x_{F_0} + \underbrace{v}_{\text{pulse speed}}\overbrace{t}^{\text{elapsed time}}$$

Another way to think about the pulse is to describe it using $f(x)$ and $f(x')$ where the peak represents y and y' respectively.

$$y(x,0) = f(x)$$

$$y(x,t) = f(x') = f(x - vt)$$

Note: $x_t = x_0 + vt$

$x_0 = x_t - vt$

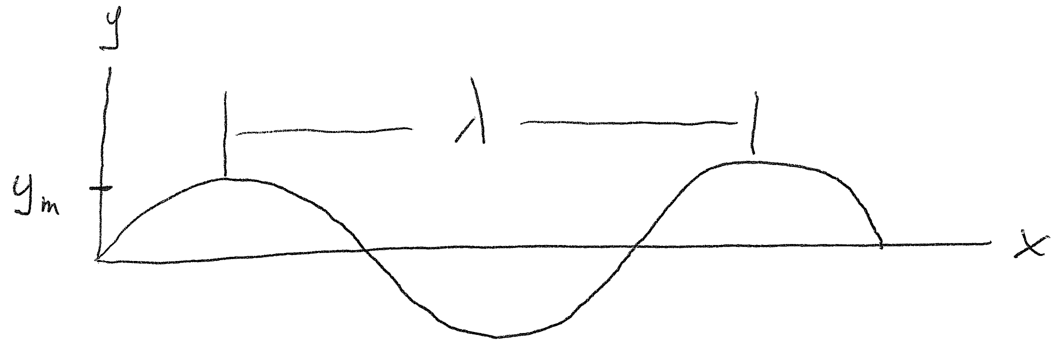
No shape change $\Longrightarrow$

$x - vt$ = constant

General Wave Equation

$$y(x,t) = f(x - vt)$$

* Any number of functions can fit this form *



$$y(x,0) = y_m \sin\left(\frac{2\pi}{\lambda} x\right)$$

As with a single pulse we can track a single point, eventually noting that we can use the general wave equation in this case.

$$y(x,t) = y_m \sin \frac{2\pi}{\lambda}(x - vt)$$

$$\lambda = vT$$

T = period (s)
v = wavespeed (m/s)
λ = wavelength (m)

$$\omega = \frac{2\pi}{T}$$

ω = angular frequency $\left(\frac{rad}{s}\right)$

$$k = \frac{2\pi}{\lambda}$$

wave number $\left(\frac{rad}{m}\right)$

$$y(x,t) = y_m \sin(kx - \omega t)$$

$$\frac{2\pi}{\lambda} x = kx$$

$$\frac{2\pi}{\lambda} vt = \frac{2\pi}{\cancel{\lambda}}\left(\frac{\cancel{\lambda}}{T}\right) t = \omega t$$