Energy in SHM

Since it has been established that a spring acts as a simple harmonic oscillator

$$\Downarrow$$

$$E_{el} = \frac{1}{2} k x^2$$

x is time dependent

$$x(t) = x_m \cos(\omega t + \phi)$$

$$\Downarrow$$

$$E_{el}(t) = \frac{1}{2} k \left[x_m \cos(\omega t + \phi) \right]^2$$

$$\boxed{= \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)}$$

$$E_k = \frac{1}{2} m v^2$$

v is time dependent.

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$\Downarrow$$

$$E_k(t) = \frac{1}{2} m \left[-\omega x_m \sin(\omega t + \phi) \right]^2$$

$$= \frac{1}{2} m \omega^2 x_m^2 \sin^2(\omega t + \phi)$$

Since $\omega = \sqrt{\frac{k}{m}} \Rightarrow \frac{k}{m} = \underline{\omega^2}$

$$= \frac{1}{2} \cancel{m} \left(\frac{k}{\cancel{m}} \right) x_m^2 \sin^2(\omega t + \phi)$$

$$\boxed{= \frac{1}{2} k x_m^2 \sin^2(\omega t + \phi)}$$

$$E_{Total} = E_k + E_{el}$$

and

$$\cos^2(\alpha) + \sin^2(\alpha) = 1$$

$$E_{Total} = \tfrac{1}{2} k x_m^2 \sin^2(\omega t + \phi) + \tfrac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$$

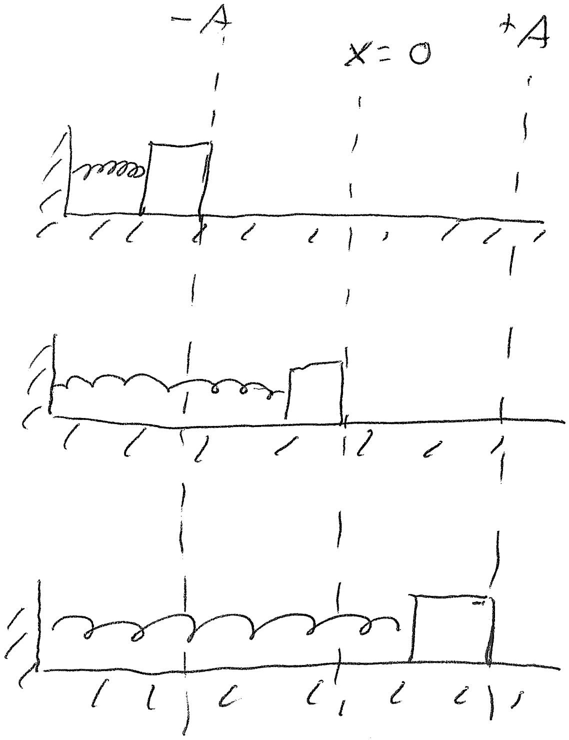
$$= \tfrac{1}{2} k x_m^2 \left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) \right]$$

$$\boxed{= \tfrac{1}{2} k x_m^2}$$

* Conceptual Reasoning

A spring-mass system that is oscillating in a frictionless environment does not have any outside forces acting.

No outside force $\Rightarrow$ No energy transfer outside of system



E_{el}

E_k

E_{el}

Circle size does not change

$\Downarrow$

Total Energy Is related to max elongatio and stiffness

$$E_{el_{max}} = E_{k_{max}} = \frac{1}{2} k x^2_{max}$$