Oscillations

Oscillation

- Movement about an equilibrium position or a starting point usually back to the starting point.

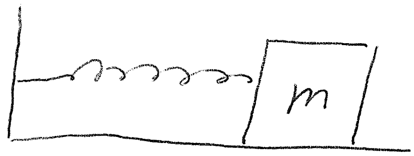
Period (T)

- time for one complete oscillation
- measured in seconds (s)

frequency (f)

- number of oscillations during a period of time
- measured in Hertz (Hz)

$$f = \frac{1}{T}$$



For a spring-mass system w/o friction

$$\Sigma F = F_S$$

$$\Rightarrow \quad F_S = F_{Net}$$

$$-kx = ma$$

$$a = \frac{d^2x}{dt}$$

$$\Rightarrow \quad -kx = m\frac{d^2x}{dt}$$

or

$$m\frac{d^2x}{dt} + kx = 0$$

Equation of Motion

This tells us that a solution to this differential equation must have the second derivative be the opposite of some constant times the function

some constant

$$\frac{d^2x}{dt} = -\left(\frac{k}{m}\right) x$$

Function

Second derivative of function

$$\underbrace{\frac{d^2}{dt} \cos \omega t}_{\text{second derivative of } \cos \omega t} = \overbrace{-\omega^2 \cos \omega t}^{\text{second derivative of function}}$$

Some constant

Function

If we let $\omega^2 = \frac{k}{m}$ then

we have a possible solution

$$x(t) = \cos \omega t$$

multiplication of $x(t)$ by a constant does not change the solution

$$x(t) = x_m \cos(\omega t)$$

adding a phase constant also does not change the solution

$$x(t) = x_m \cos(\omega t + \phi)$$

This is merely a shift in the starting position.

Simple Harmonic Motion

- periodic motion that obeys the following equations

$$x(t) = x_m \cos(\omega t + \phi)$$

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

where

$a(t) = -\omega^2 x(t)$

x_m = maximum displacement

* at a specific time

$a = -\omega^2 x$

ω = angular frequency

ϕ = phase constant

are constants.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

SHM Force Law

- SHM is the motion produced by a particle that is acted upon by a force that is proportional to displacement, but opposite in sign.

Since

$F = ma$ $\qquad\qquad$ $F(t) = m\,a(t)$

$a = -\omega^2 x$ $\qquad\qquad$ $a(t) = -\omega^2\, x(t)$

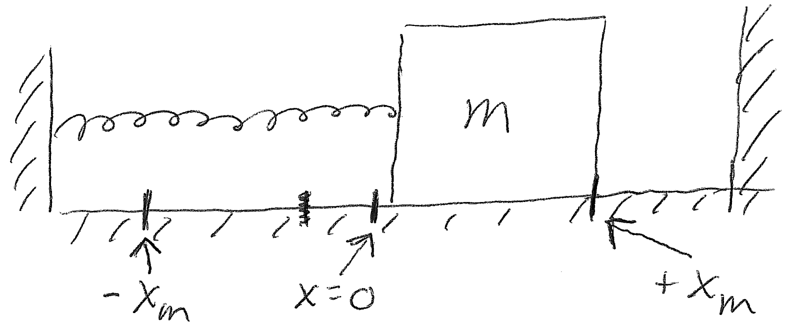
$F = m(-\omega^2 x)$ $\qquad\qquad$ $F(t) = m\,-\omega^2 x(t)$

$= -m\omega^2 x$ $\qquad\qquad$ $= -m\omega^2\, x(t)$

Note the similarity

$F = -kx \implies k = m\omega^2$

Spring - Mass System



The system above is a simple harmonic oscillator because the net force acting on the block is $F = -kx$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

$$x(t) = x_m\left(\cos\left(\sqrt{\frac{k}{m}}\,t\right)\right) \quad \text{for } \phi = 0$$

$$v(t) = -\left(\sqrt{\frac{k}{m}}\right) x_m \sin\left(\sqrt{\frac{k}{m}}\,t\right)$$

$$a(t) = -\left(\frac{k}{m}\right) x_m \cos\left(\sqrt{\frac{k}{m}}\,t\right)$$