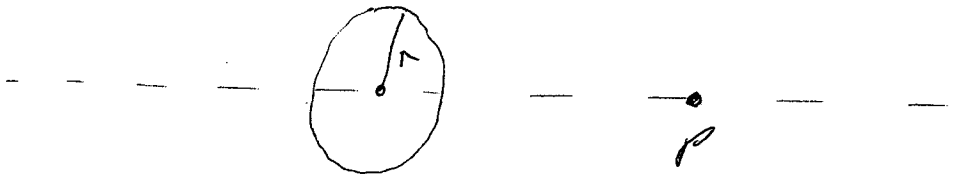
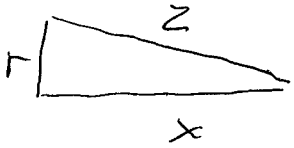
Determine the gravitational field strength at a point P that is a distance x away from the center of the ring. Suppose the ring is uniform. $(\lambda = \frac{M}{L})$





$dM = \lambda\, dl$ Note: dl is an arc length

$dl = r\,d\theta$

$dM = \lambda r\, d\theta$

$$\frac{dF}{m} = \frac{G\,dM}{r^2}$$ Note: $r^2 = z^2$

$$\frac{dF}{m} = \frac{G\lambda r\, d\theta}{z^2}$$ Note: all y-components cancel by symmetry

$$\frac{dF}{m_x} = \frac{G\lambda r\, d\theta}{z^2}\cos\phi \qquad \cos\phi = \frac{adj}{hyp} = \frac{x}{z}$$

$$= \frac{G\lambda r\, d\theta}{z^2}\,\frac{x}{z}$$

$$\frac{dF}{m_x} = \frac{G\lambda r x}{z^3}\, d\theta$$

$$\frac{F}{m_x} = \frac{G\lambda r x}{z^3}\int_0^{2\pi} d\theta$$

$$= \frac{G\lambda r x}{z^3}\, 2\pi \qquad \lambda = \frac{M}{L} = \frac{M}{2\pi r}$$

$$\frac{F}{m} = \frac{G\cancel{\lambda}}{z^3}\,\frac{M}{\cancel{2\pi r}}\,\cancel{2\pi r} = \boxed{\frac{GMx}{z^3}}$$

or

$$\boxed{\frac{F}{m} = GM\left(\frac{x}{(r^2+x^2)^{3/2}}\right)}$$