Combined Translation and Rotation

As long as no slipping occurs

$V = R\omega$ can be used to relate angular and linear velocity values.

Kinetic energy must also reflect both movements.

$$E_k = E_{k_{rotational}} + E_{k_{linear}}$$

analogous velocity

$$E_k = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

analogous mass

This means velocity can be expressed as v or ω as long as no slipping occurs.

$$E_k = \frac{1}{2} I \omega^2 + \frac{1}{2} m (\omega r)^2$$

or

$$E_k = \frac{1}{2} I \left(\frac{v}{r}\right)^2 + \frac{1}{2} m v^2$$

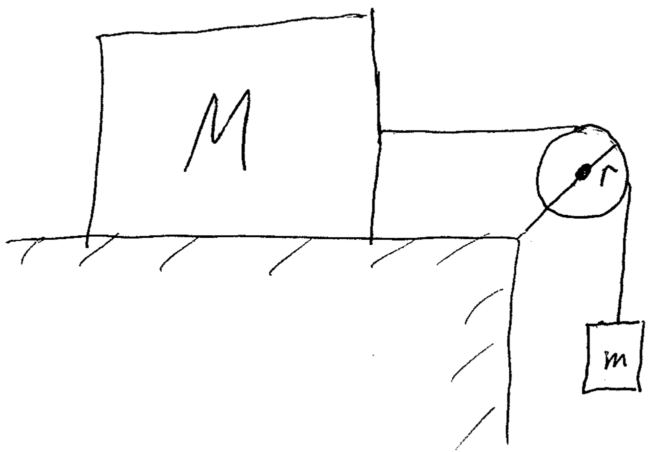
Accelerations can also be related when no slipping occurs $\quad a = \alpha r$

$$\tau = I\alpha \qquad F = ma$$

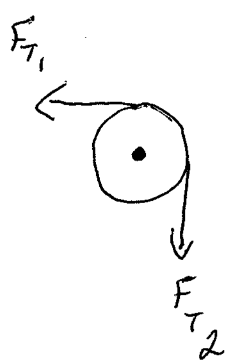
$$\downarrow \qquad\qquad \downarrow$$

$$\tau = I\left(\frac{a}{r}\right) \qquad F = m\alpha r$$

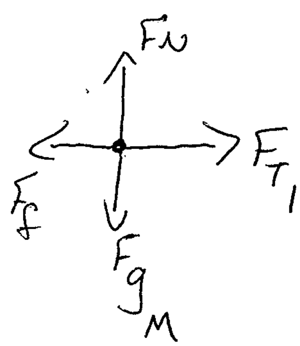
This can be used when translating objects are coupled with rotating objects.



Pulley



M



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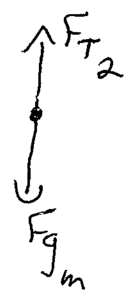

$$\sum \tau = I\alpha = F_{T_1} r + -F_{T_2} r$$

$$\sum F_x = -F_f + F_{T_1} = Ma$$

$$\sum F_y = ma = F_{T_2} - mg$$

$$F_{T_1} = Ma + F_f$$

$$F_{T_2} = ma + mg$$

$$I\alpha = (Ma + F_f)r + -(ma + mg)r$$

Depending on what you want

$\alpha \rightarrow a$

or

$a \rightarrow \alpha$

can be used to solve for the respective system acceleration.

If a is desired

$$I\left(\frac{a}{r}\right) = (Ma + F)r + -(ma + mg)r$$

If α is desired

$$I\alpha = (M\alpha r + F)r + -(m\alpha r + mg)r$$