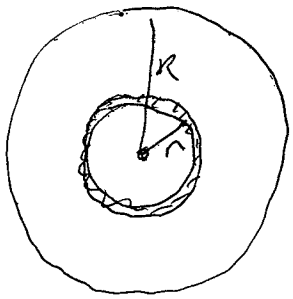
Determine the rotational inertia (I) of a disk that has mass distributed uniformly throughout its volume (ρ).



$$I_{ring} = mr^2$$

Break disk into rings and sum all contributions from $0 \to R$

$$dI_{disk} = dm_{disk}\, r^2 \qquad \rho = \frac{m}{V} \qquad V = \pi r^2 h$$

$$dm = \rho\, dV \qquad dV = 2\pi h\, r\, dr$$

$$dm = \rho 2\pi h\, r\, dr$$

$$I_{disk} = \int_0^R \rho 2\pi h\, r\, dr\, r^2 = \rho 2\pi h \int_0^R r^3\, dr$$

$$= \rho 2\pi h \frac{R^4}{4}$$

$$I_{disk} = \rho \cancel{2}\pi h R^2 \frac{R^2}{\cancel{4}_{2}}$$

$$= (\rho \pi h R^2) \frac{R^2}{2}$$

$$m = \rho V$$

$$= \rho \pi R^2 h$$

$$\boxed{I_{disk} = \frac{m R^2}{2}}$$